Observational evidence for exponential tornado intensity distributions over specific kinetic energy

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[1] Observational evidence supports the recent analytical prediction that tornado intensities are exponentially distributed over peak wind speed squared (v^2), or equivalently, Rayleigh-distributed over v. For large USA data samples, exponential tails are found in the tornado intensity distributions over v^2 from about F2 intensity on. Similar results follow for smaller worldwide data samples. For the 1990s data from the USA and Oklahoma, deviations from the Rayleigh distribution for weak tornadoes can be explained by the emergence of a separate, likely non-mesocyclonic tornado mode. These bimodal datasets can be modeled by superposition of two Rayleigh distributions. The change in modal dominance occurs at about the F2 threshold ($v \approx 50$ m s^{-1}). In France, likely mainly the mesocyclonic tornado mode has been recorded, while in the UK, only a non-mesocyclonic mode seems to be present. **Citation:** Dotzek, N., M. V. Kurgansky, J. Grieser, B. Feuerstein, and P. Névir (2005), Observational evidence for exponential tornado intensity distributions over specific kinetic energy, *Geophys. Res. Lett.*, *32*, L24813, doi:10.1029/2005GL024583.

1. Introduction

[2] Tornadoes are low-risk-high-impact phenomena, and it is of vital socio-economic interest to model tornado intensity distributions as well as their temporal development. Usually no direct wind speed measurements are available from within a tornado, but its damage is related to its maximum velocity or a power of the peak winds. Commonly, the Fujita scale (F-scale) [e.g., *Fujita*, 1981] is applied to link maximum damage to wind speed classes:

$$v(\mathrm{F}) = 6.30\ \mathrm{m\,s^{-1}}(\mathrm{F}+2)^{3/2}. \quad (1)$$

[3] However, the maximum horizontal wind speed v (or momentum density) of a tornado, its maximum values of kinetic energy ($\propto v^2$) or energy-flux density ($\propto v^3$) bear more physical relevance than the F-scale:

$$v = \frac{M}{\rho}, \quad \frac{2E_{kin}}{\rho} = v^2 = \frac{2\Delta p_s}{\rho}, \quad v^3 = \frac{2P_{kin}}{\rho}. \quad (2)$$

As equation (2) shows, v is coupled to the specific values of mass flux M, kinetic energy E_{kin}, stagnation pressure difference Δp_s, and energy flux density P_{kin} (ρ denotes air density). Depending on structural characteristics, v^2 or v^3 are related to wind load and damage.

[4] *Brooks and Doswell* [2001] discussed exponential tornado intensity distributions over F-scale. However, these would lead to an overwhelming estimated number of apparently unobserved weak or sub-critical tornadoes. Besides, an exponential in F could not account for any upper limit in tornado horizontal wind speeds following from energy budget considerations [cf. *Dotzek et al.*, 2003].

[5] As worldwide tornado intensity distributions consistently deviate from exponentials in F-scale, *Dotzek et al.* [2003] did a more thorough statistical analysis of these observations. They found three-parameter Weibull distributions in either wind speed v or F-scale to be adequate. Furthermore, the two free fit parameters c and b (see equation (5)) of different regions displayed dependence due to a common property which was identified by *Feuerstein et al.* [2005] as a constant ratio of strong to violent tornado reports in consecutive intensity classes. Yet, no physical reason for this universal feature could be given.

[6] Based on very general thermodynamic and fluid dynamical assumptions, *Kurgansky* [2000] deduced an exponential distribution of v^2 in tornadoes, i.e., a Rayleigh distribution for the maximum wind speed v. He compared it with observed tornado intensities over the territory of the former USSR [*Snitkovsky*, 1987] by estimating the single free parameter of the Rayleigh distribution from the second moment of the observations. Comparison of the resulting distribution with the observations was successful. Also, general agreement between 1950–1994 USA tornado statistics and the reference Rayleigh distribution was found. Yet, it had not been tested if tornadoes worldwide follow the analytically predicted distribution. This is the subject of the present paper.

2. Analytical Framework

2.1. Rayleigh Distribution

[7] *Kurgansky* [2000] considered a statistical ensemble of vertical air columns of unit horizontal cross-section, in whom (i) a helical motion is embedded, (ii) the Beltrami

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flow condition holds, and (iii) the total helicity H is invariant. Herein, an H-based probabilistic measure and the corresponding canonical negative exponential distribution were introduced. From this, *Kurgansky* [2000] derived an exponential probability density function p of tornado intensities as a function of the maximum swirl velocity squared, v^2:

$$p(v^2) = \frac{dP(v^2)}{dv^2} = \frac{1}{v_0^2}\exp\left(-\frac{v^2}{v_0^2}\right). \quad (3)$$

After a transformation of variables, the same distribution, expressed as $p(v)$ instead of $p(v^2)$, becomes a Rayleigh distribution and reads:

$$p(v) = \frac{dP(v)}{dv} = \frac{dP(v^2)}{dv^2}\frac{dv^2}{dv} = \frac{2v}{v_0^2}\exp\left(-\frac{v^2}{v_0^2}\right). \quad (4)$$

2.2. Weibull Distribution

[8] *Dotzek et al.* [2003] and *Feuerstein et al.* [2005] assumed Weibull distributions and applied a two-parameter least-square fit to observed worldwide tornado intensity distributions both in F and v. With x denoting either of these, the Weibull distribution is given in three-parameter form for probability $P(x)$ and probability density $p(x) = dP/dx$:

$$P(x) = 1 - \exp\left(-\left(\frac{x-a}{b}\right)^c\right);$$
$$p(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1}\exp\left(-\left(\frac{x-a}{b}\right)^c\right). \quad (5)$$

[9] Here, a is a fixed parameter and denotes the lower boundary of the variable x. The scaling factor b and the shape parameter c are the two model parameters to be estimated. Note that for $c = 1$, equation (5) reduces to an exponential distribution, whereas $c = 2$ yields the Rayleigh distribution:

$$p(x) = \frac{dP}{dx} = \frac{2(x-a)}{b^2}\exp\left(-\frac{(x-a)^2}{b^2}\right). \quad (6)$$

As the remainder of this paper deals exclusively with wind speed v as the independent variable, we substitute $v \equiv x - a$, and $v_0 \equiv b$. Then, equations (4) and (6) coincide identically.

2.3. Practical Reporting Issues

[10] Tornadoes are usually reported only above the lower F0 threshold of $v_{\rm min}$ = 17.82 m s^{-1} and below the upper F5 threshold of $v_{\rm max}$ = 142.55 m s^{-1} (see equation (1)). To properly normalize the probability density functions, such lower and upper bounds have to be taken into account, because the norm N of the Rayleigh distribution

$$N = \int_{v_{\rm min}}^{v_{\rm max}} \frac{2v}{v_0^2}\exp\left(-\frac{v^2}{v_0^2}\right)dv = \exp\left(-\frac{v_{\rm min}^2}{v_0^2}\right) - \exp\left(-\frac{v_{\rm max}^2}{v_0^2}\right) \quad (7)$$

differs from unity for $v_{\rm min} > 0$ or finite $v_{\rm max}$, and equations (4) and (6) have to be divided by N to ensure correct normalization.

[11] The second moment of the Rayleigh distribution for arbitrary values $v_{\rm min}$ and $v_{\rm max}$ as in equation (7) is given by:

$$\langle v^2\rangle_{v_{\rm min}}^{v_{\rm max}} = v_0^2 - \frac{1}{N}\left[v_{\rm max}^2\exp\left(-\frac{v_{\rm max}^2}{v_0^2}\right) - v_{\rm min}^2\exp\left(-\frac{v_{\rm min}^2}{v_0^2}\right)\right]. \quad (8)$$

Iterative solution of the implicit equation (8) for v_0 converges very rapidly. For the case $v_{\rm max} \to \infty$, one obtains the expression given by *Kurgansky* [2000]:

$$\langle v^2\rangle_{v_{\rm min}}^{\infty} = v_0^2 + v_{\rm min}^2. \quad (9)$$

[12] *Kurgansky* [2000] took into account the full dataset on tornado intensities provided by *Snitkovsky* [1987]; that is, he included all observations from F0 to F4 on the Fujita scale. However, *Dotzek et al.* [2003] demonstrated that considering tornado reports only from F1 upward leads to an enhanced fit quality and a more realistic estimate of the likely underreporting of F0 tornadoes. By adjusting $v_{\rm min}$ from v(F0) to v(F1), the moment estimation technique also allows to exclude the F0 events.

3. Observational Evidence

[13] *Kurgansky* [2000] applied his moment estimation technique to the *Snitkovsky* [1987] data for the territory of the former USSR, and a Rayleigh distribution corresponded well to the observed tornado probability densities over F-scale. *Dotzek et al.* [2003] used slightly different data from the same region, and their two-parameter Weibull fit in v led to an exponent of c = 2.119 and $b = v_0$ = 42.090 m s^{-1}. The c-value is indeed very close to the predicted value of 2, and also v_0 corresponds fairly well to Kurgansky's estimate of 39.473 m s^{-1}. Using exactly the *Snitkovsky* [1987] data, the two-parameter Weibull estimate leads to c = 1.892 and v_0 = 38.048 m s^{-1}. So, a Rayleigh distribution appears to be justified here.

[14] *Dotzek et al.* [2003] also performed the Weibull fits in v for a large number of datasets worldwide. *Dotzek et al.* [2003, Table 3] show that most c-values lie in the range from 1.6 to 2.5, with a general trend from higher to lower values with improving data quality and sample size over time. Only the data for the USA and Oklahoma from the 1990s produce c-values closer to unity: c = 1.157 and 1.207, respectively. The other USA data support $c \approx 2$. The data for regions outside the USA [*Dotzek et al.*, 2003, Table 4] showed values of c ranging from about 1.5 to more than 3.5. Here, data quality issues in the much smaller non-USA data samples play a role, and apparently, the larger scatter in c-values of these data may have kept *Dotzek et al.* [2003] from concluding any relevance to c = 2.

[15] However, rather compelling evidence for c = 2, i.e., exponential distributions over v^2 or Rayleigh distributions over v, is found in a graphical display (Figure 1). The USA data in Figure 1a have the largest sample sizes and show a very clear exponential distribution over v^2 for the tornadoes of more than F1 intensity. Over time, the probabilities of weak tornadoes approach this exponential law from below

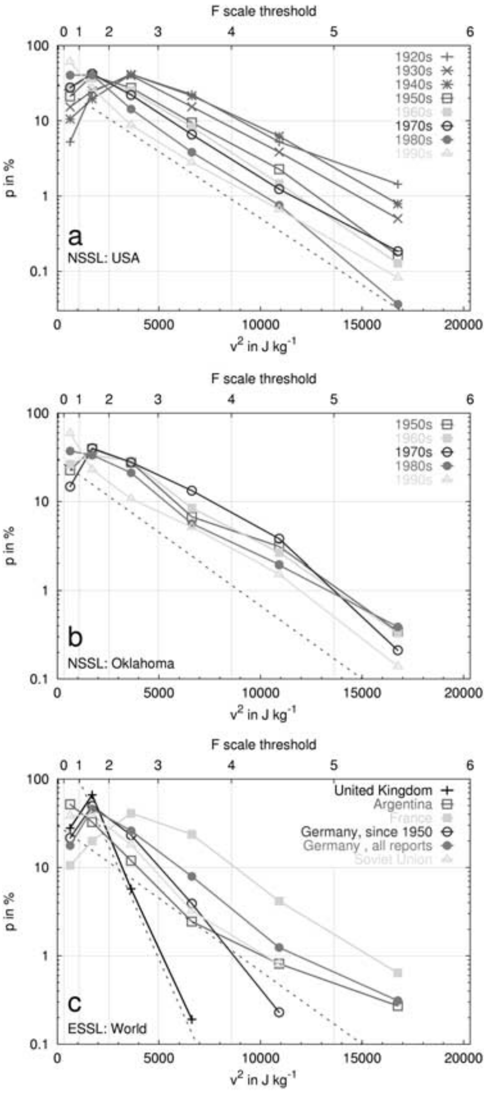

Figure 1. Tornado intensity distributions over v^2; (a) USA, decadal from 1920 to 1999, (b) Oklahoma, decadal from 1950 to 1999, (c) worldwide, with the dashed line from (b) for comparison. See color version of this figure in the HTML.

and only exceed it in the 1990s. The same observations follow from the Oklahoma data in Figure 1b. Individual sample size is smaller here, and the slope of the distributions is not as uniform as for the whole USA. Nevertheless, the distributions still follow an exponential distribution in v^2 as before, for tornadoes of at least F1 intensity.

[16] The worldwide data in Figure 1c, including those of *Snitkovsky* [1987], offer some interesting insights. Here, the sample size is smaller than, or at most comparable to, the Oklahoma data, and internal variability in the distribution must be larger than for the previously shown data. Especially, the number of F5 tornado observations is very small (1 or 2). From the 1990s USA data, probability of F5 tornadoes appears to be less than 0.1%. As the data in Figure 1c contain one or two historical F5 reports and the sample size is only some hundreds, the observationally resolved probability of F5 events must be considerably larger than the expected climatological value. This is the case with the data from Germany (all events) and Argentina, leading to spurious leftward curvature at high intensities.

[17] Taking these limitations into account, evidence for exponential tails in the distributions of Figure 1c can still be diagnosed. This holds both for France with a high percentage of stronger tornadoes being reported, for Germany with a slope comparable to that of US-tornadoes (dashed line), and also for the United Kingdom, from where a vast majority of weak tornadoes is reported.

[18] The UK data are especially interesting. The two-parameter Weibull fit in v by *Dotzek et al.* [2003] had shown a value of c = 3.833 which did not point to a Rayleigh distribution. Now, Figure 1c clearly reveals a nearly perfect exponential tail in the distribution from F1 on. Apparently, only the inclusion of the low number of reported F0 tornadoes together with the steep slope of the exponential tail had led to the high c-value in the Weibull fit. Neglecting the F0 observations, c = 1.510 is derived, much closer to 2. So there is worldwide evidence for the validity of Rayleigh distributions in v.

[19] What remains to be clarified is why the F0 (and to some degree the F1) tornadoes in the 1990s USA and Oklahoma data have exceeded the exponential slope in Figure 1. This can be explained by an evolution toward bimodal distributions [cf. *Feuerstein et al.*, 2005]. Figure 2 again shows the 1990s data for (a) the USA, and (b) Oklahoma, but now as a function of maximum wind speed v instead of v^2. Apparently, the data can be reproduced by a superposition of two Rayleigh distributions with different v_0-values: v_{01} = 25.0 m s^{-1}, v_{02} = 49.5 m s^{-1} for the USA, and v_{01} = 21.8 m s^{-1}, v_{02} = 51.3 m s^{-1} for Oklahoma.

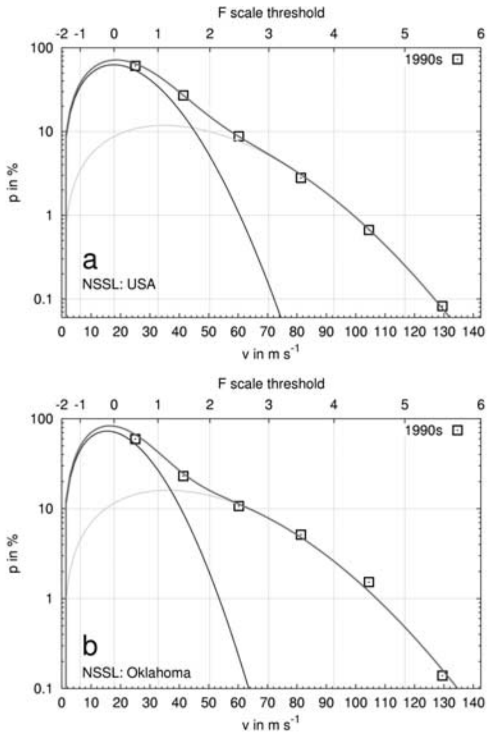

Figure 2. Tornado intensity data (boxes) from (a) USA, (b) Oklahoma in the 1990s. The curves show two Rayleigh distribution modes and their superposition. See color version of this figure in the HTML.

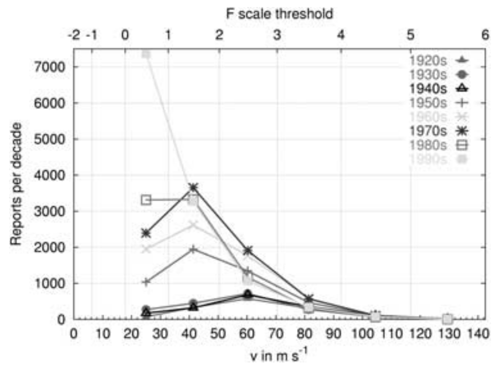

Figure 3. Number of reported tornadoes per F-scale class in the USA, decadal from 1920 to 1999. See color version of this figure in the HTML.

[20] Figure 2 also shows that while the two individual modes of tornadoes can be well-described by two Rayleigh distributions, the sum of these two distributions is no longer a true Rayleigh distribution, but shows some leftward curvature at low intensities in a lin-log diagram. Only from about F2 intensity on, the likely mesocyclonic mode dominates such that the right tail of the distribution follows the Rayleigh distribution of equations (4) and (6). This explains why *Dotzek et al.* [2003] diagnosed Weibull exponents c considerably below 2 for the 1990s USA and Oklahoma data when all reports from F0 onward were taken into account. When the F0 tornadoes are eliminated from the two-parameter Weibull fits in v, larger values for c are derived, for example,. for Oklahoma, $c = 1.7$ instead of $c = 1.2$. From Figure 2, c-values even closer to 2 can be expected when only data from F2 onward are used.

4. Discussion

[21] Our methodology relies on at least on-average accuracy of F-scale ratings. Yet, it is well known that even experienced damage surveyors can disagree on the F-scale for a particular structure. Further, for example, *Verbout et al.* [2006] note that prior to the adoption of the F-scale, US tornadoes from 1950 to 1973 were systematically overrated a posteriori (see Figure 3).

[22] However, our results are unlikely to be critically affected by these issues: First, uncertainties in F-scale assignments can be both toward higher and lower intensity, so assignment errors should tend to cancel out from about F1 to F4 intensity. Second, despite probable overrating from 1950 to 1973, Figure 1a shows that the distributions from that period are still exponential, and fit into the trend of reporting lower percentages of significant tornadoes over time.

[23] Tornado intensity distributions may be bimodal if estimated c-values of a Weibull fit drop significantly below the predicted value of $c = 2$. And for likely bimodal distributions, it might be desirable to neglect the F0 and even F1 observations when the goal is to model the high intensity tail of the distributions. Vice versa, it may not be advisable to include the reports stronger than F2 when modeling the weak part of the intensity spectrum.

[24] Following the arguments given by *Feuerstein et al.* [2005], the two separate modes likely can be attributed to non-mesocyclonic and mesocyclonic tornadoes, respectively. In earlier decades, presumably mainly the stronger mesocyclonic tornadoes have been recorded. From the 1950s on, the number of tornado reports has increased tremendously in the USA (Figure 3). However, the increase only affected the number of F0 to F2 tornadoes, and this is exactly the intensity range of the non-mesocyclonic events.

[25] The 1990s with large scientific field programs (e.g., VORTEX) and widespread public awareness made it likely that so many non-mesocyclonic tornadoes have been observed and reported that the lower mode in the intensity distributions could emerge. So, if reporting more tornadoes mainly means reporting more non-mesocyclonic events, then it is plausible that the spectrum transition from mode 1 to mode 2 in Figure 2 occurs somewhere in the F2 intensity class. Interestingly, also *Brooks and Doswell* [2001] had fixed the number of F2 reports when comparing different tornado intensity distribution slopes.

[26] Further information can be drawn from Figure 1c: In France, likely only the mesocyclonic tornado mode has been recorded, while in the UK, only a non-mesocyclonic mode with v_0 close to 25 m s^{-1} appears to be present. The other distributions have relatively similar slopes, which also bear some resemblance to that inferred from the USA data (dashed line). That is, these distributions are certainly dominated by reports of mesocyclonic events, and aside from the data quality issues mentioned above, their variations in slope point to a different percentage of non-mesocyclonic events being reported.

5. Conclusions

[27] Our study showed:

[28] • There is strong evidence worldwide for exponential tornado intensity distributions in v^2, or equivalently, Rayleigh distributions in v. Datasets with a high percentage of recorded weak tornadoes (F0, F1) can be modeled by a superposition of two individual Rayleigh-distributed modes.

[29] • Exponential and Rayleigh distributions are special cases of the Weibull distribution, so a two-parameter Weibull fit is an appropriate way to detect and quantify deviations from the Rayleigh distribution, caused either by detection efficiency issues or by emerging bimodality.

[30] • Likely bimodality in tornado intensity distributions manifests itself in a two-parameter Weibull fit by exponents c significantly smaller than two, for example, $c < 1.3$, if caused by leftward curvature from F0 to F2 intensity in a lin-log diagram.

[31] • As the spectral dominance changes from the non-mesocyclonic to the mesocyclonic mode at about the F2 threshold ($v = 50.4$ m s^{-1}), it seems advisable to model mode 1 using mainly the F0 and F1 observations, and mode 2 only based on F2 to F5 observations.

[32] Full synthesis of our two statistical modeling approaches is subject of a forthcoming paper.

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